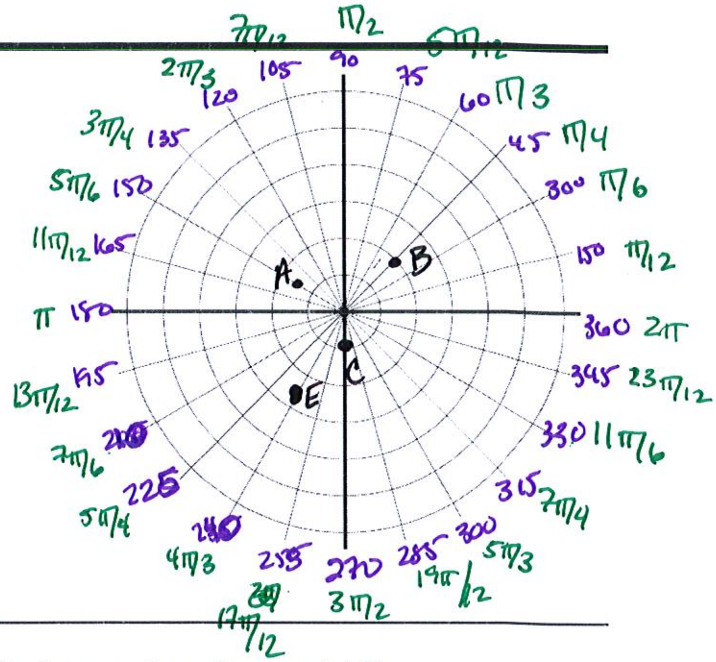
Review Answers

GSE PreCalculus

Unit 8: Polar Functions Review

Graph the following polar points:

1. A $\left(1.5, -\frac{7\pi}{6}\right)$ $\quad -\frac{7\pi}{6} + 2\pi = 5\pi/6$
2. B $\left(-2, -135^\circ\right)$
3. C $\left(-1, \frac{\pi}{2}\right)$
4. E $\left(2.5, 240^\circ\right)$



Find four different pairs of polar coordinates that name the given point if:

$-360^\circ \le \sigma \le 360^\circ$ or $-2\pi \le \sigma \le 2\pi$

5. $\left(2, -150^\circ\right)$ $\quad (2, 210^\circ)$, $(-2, 30^\circ)$, $(-2, -330)$
6. $\left(5, 240^\circ\right)$ $\quad (5, -120)$, $(-5, 60)$, $(-5, -300)$
7. $\left(2, \frac{\pi}{6}\right)$ $\quad (2, -11\pi/6)$, $(-2, 7\pi/6)$, $(-2, -5\pi/6)$

$(x, y) \longrightarrow (r, \theta)$

Rectangular Coordinates to Polar Coordinates:

$r^2 = x^2 + y^2 \qquad r = \sqrt{x^2+y^2}$

$x = r\cos\theta \qquad y = r\sin\theta$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$a > 0$

$a < 0 \; +180$

8. (8,10) $\quad r = \sqrt{8^2 + 10^2} \quad \theta = \tan^{-1}\left(\frac{10}{8}\right)$

$r \approx 12.8 \quad \theta = 51.3$

$\boxed{\text{so, } (12.8, 51.3^\circ)}$

9. (-9, -4) $\quad r = 9.85 \quad \theta = \tan^{-1}\left(\frac{-4}{-9}\right) + 180$

$r = \sqrt{(-9)^2 + (-4)^2}$

$= \sqrt{81 + 16}$

$\theta = 204^\circ$

$\boxed{(9.85, 204^\circ)}$

$(r, \theta) \longrightarrow (x, y)$

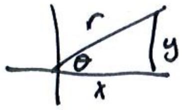
Polar Coordinates to Rectangular Coordinates:

10. (3, -120°) $\quad x = 3\cos -120^\circ \quad y = 3\sin -120^\circ$

$\left(\frac{-3}{2}, \frac{-3\sqrt{3}}{2}\right)$ or $\boxed{(-1.5, -2.6)}$

11. (-2, 135°) $\quad x = -2\cos 135 \quad y = -2\sin 135$

$(\sqrt{2}, -\sqrt{2})$ or $\boxed{(1.4, -1.4)}$



$\cos\theta = \frac{x}{r}$
$\sin\theta = \frac{y}{r}$
$\tan\theta = y/x$

$r^2 = x^2 + y^2$ $\quad r = \sqrt{x^2+y^2}$
$x = r\cos\theta$ $\quad y = r\sin\theta$
$\theta = \tan^{-1}\left(\frac{b}{a}\right)$
$a<0$ $+180$

Unit 8: Polar Functions Review

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1. Convert from rectangular equations to polar equations:

→ change to + **

12. $y = \sqrt{3}x$

$\frac{y}{x} = \sqrt{3}$

$\tan^{-1}(\tan)\theta = (\sqrt{3})\tan^{-1}$

$\boxed{\theta = 60^\circ}$

13. $x^2 + (y-3)^2 = 9$

$x^2 + (y-3)(y-3) = 9$

$x^2 + y^2 - 6y + 9 = 9$
$\quad -9 \quad -9$

$x^2 + y^2 - 6y = 0$
$\quad +6y \quad +6y$

$x^2 + y^2 = 6y$

$\frac{r^2}{r} = \frac{6(r\sin\theta)}{r}$

$\boxed{r = 6\sin\theta}$

14. $x^2 + y^2 = 1$

$\sqrt{r^2} = \sqrt{1}$

$\boxed{r = 1}$

Convert from polar equations to rectangular equations. Then, identify the resulting figure.

* Change calculator to radians!

13. $r = 10$

$\left(\sqrt{x^2+y^2}\right)^2 = (10)^2$

$\boxed{x^2 + y^2 = 100}$

Circle: ctr $(0,0)$
r: 10

14. $\theta = -\frac{\pi}{3}$

$\tan\theta = \tan\left(-\pi/3\right)$

$x\left(\frac{y}{x}\right) = (-\sqrt{3})x$

$\boxed{y = -\sqrt{3}x}$

line: $m = -\sqrt{3}$
$b = 0$

15. $r = 2\cos\theta$

$r(r) = (2\cos\theta)r$

$r^2 = 2r\cos\theta$

$x^2 + y^2 = 2x$
$-2x \quad -2x$

$x^2 - 2x + y^2 = 0$

* Complete the square!

$\left(\frac{-2}{2}\right)^2$

$x^2 - 2x + 1 + y^2 = 1$

$\boxed{(x-1)^2 + y^2 = 1}$

Circle: ctr $(1,0)$
$r = 1$

16. $(r) = \left(\frac{1}{\cos\theta + \sin\theta}\right)(\cos\theta + \sin\theta)$

$r(\cos\theta + \sin\theta) = 1$

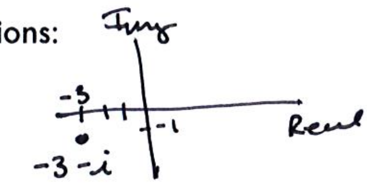
$r\cos\theta + r\sin\theta = 1$

$\boxed{x + y = 1}$

line: $m = -1$
$b = 1$

Represent complex numbers (polar form) & complex number operations:

17. Explain how you would represent $-3 - i$ on the complex plane.



18. Find the conjugate of $-4 + 2i$. Conjugate: $-4 - 2i$

19. $\frac{6-i}{-4+2i}$

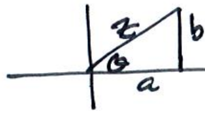
$= \boxed{-1.3 - .4i}$

20. $(-1+4i) - (2+7i)$

$\boxed{-3 - 3i}$

21. $(2-i)(3+4i)$

$\boxed{10 + 5i}$



$\cos\theta = \frac{a}{z}$
$\sin\theta = \frac{b}{z}$
$\tan\theta = b/a$

$z^2 = a^2 + b^2$ $z = \sqrt{a^2+b^2}$
$a = r\cos\theta$ $b = r\sin\theta$
$\theta = \tan^{-1}\left(\frac{b}{a}\right)$
$a<0$ $+180°$

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$r(\cos\theta + i\sin\theta)$

$a+bi \rightarrow (a,b) \rightarrow (r,\theta)$

Express each complex number in polar form:

22. $-2+5i$ $(-2,5)$ a, b

$r = \sqrt{(-2)^2+(5)^2}$
$= \sqrt{29}$
$= 5.4$

$\theta = \tan^{-1}\left(\frac{5}{-2}\right) + 180°$
$\theta = 111.8$

$\boxed{\text{So, } 5.4(\cos 111.8 + i\sin 111.8)}$

23. $6+2i$ $(6,2)$ $r = \sqrt{(6)^2+(2)^2}$
$= 2\sqrt{10}$
≈ 6.3

$\theta = \tan^{-1}\left(\frac{2}{6}\right)$
$\theta = 18.4$

So, $\boxed{6.3(\cos 18.4 + i\sin 18.4)}$

Convert the polar form of a complex number to its rectangular form:

24. $z = 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$4\left(1/2 + i\frac{\sqrt{3}}{2}\right)$ $\boxed{2+2\sqrt{3}i}$

25. $z = 5\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$5\left(\frac{-\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$

$\boxed{-\frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}i}$

Cumulative Review:

26. Solve: $x - 2y = 7$
$4x + 5y = -2$

$\underbrace{\begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}}_{A}\begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} 7 \\ -2 \end{bmatrix}}_{B}$ $= A^{-1}B$

$\begin{bmatrix} 2.4 \\ -2.3 \end{bmatrix}$ or $\begin{bmatrix} 31/13 \\ -30/13 \end{bmatrix}$

omit ~~27.~~ Find the standard form equation of $\frac{2x^2}{2} - \frac{4y^2}{2} - \frac{6x}{2} + \frac{8y}{2} - \frac{10}{2} = \frac{2}{2}$ $(x^2-3x) - (2y^2+4y) = 5$

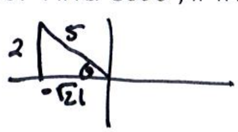
$x^2 - 2y^2 - 3x + 4y - 5 = 0$

28. Evaluate: $\sin\left(\frac{4\pi}{3}\right)$ $= -\frac{\sqrt{3}}{2}$

29. Evaluate: $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ or $150°$

30. Find $\sec\theta$, if the angle is in Quadrant II, $\sin\theta = \frac{2}{5}$

$2^2 + b^2 = 5^2$
$4 + b^2 = 25$
$\sqrt{b^2} = \sqrt{21}$
$b = \sqrt{21}$



$\cos\theta = -\frac{\sqrt{21}}{5}$ $\sec\theta = \frac{5}{-\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = -\frac{5\sqrt{21}}{21}$

31. Evaluate $\begin{vmatrix} -5 & 1 \\ -2 & -2 \end{vmatrix}$ $= 12$

$10 - (-2) = \boxed{12}$

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32. Find the asymptotes of $y = \tan(2x - 60)$

$2x - 60 = -90$
$+60 \quad +60$
$2x = -30$
$x = -15$

$2x - 60 = 90$
$+60 \quad +60$
$\frac{2x}{2} = \frac{150}{2}$
$x = 75$

$\boxed{\text{left asymptote: } -15 \quad \text{Right asymptote: } 75}$

33. Solve: $4\sin^2 x - 3 = 0$

$4\sin^2 x - 3 = 0$
$+3 \quad +3$
$\frac{4\sin^2 x}{4} = \frac{3}{4}$

$\sqrt{\sin^2 x} = \sqrt{\frac{3}{4}}$

$\sin x = \pm \frac{\sqrt{3}}{2}$

$\boxed{x = 60° \text{ or } 300°}$

34. Find $< C$, given that $a = 6, b = 2, c = 5$

~~Cos~~ $C = \cos^{-1}\left(\frac{c^2 - a^2 - b^2}{-2ab}\right)$ $C = \cos^{-1}\left(\frac{5^2 - 6^2 - 2^2}{-2(6)(2)}\right)$

$= 51.3°$

35. Find the component form of the vector, *given* $||5||, \emptyset = 48°$

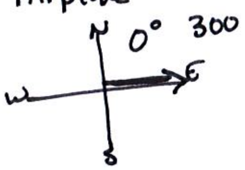
$\langle ||v|| \cos\theta, ||v|| \sin\theta \rangle$

$\langle 5\cos 48, 5\sin 48 \rangle$

$\boxed{\langle 3.3, 3.7 \rangle}$

36. An airplane is traveling 300 kilometers per hour due east. A wind is blowing 35 kilometers per hour S 75°W. What is the resulting velocity of the airplane?

Airplane



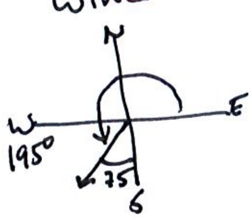
$\langle 300\cos 0, 300\sin\theta \rangle$ $\langle 300, 0 \rangle$

$+$

$\langle 35\cos 195, 35\sin 195 \rangle$ $\langle -33.8, -9.1 \rangle$

Resultant vector $\langle 266.2, -9.1 \rangle$

Wind



35

magnitude: $\sqrt{(266.2)^2 + (-9.1)^2}$
$= 266.4$ Kph

Direction: $\theta = \tan^{-1}\left(\frac{-9.1}{266.2}\right)$
$= -1.96$