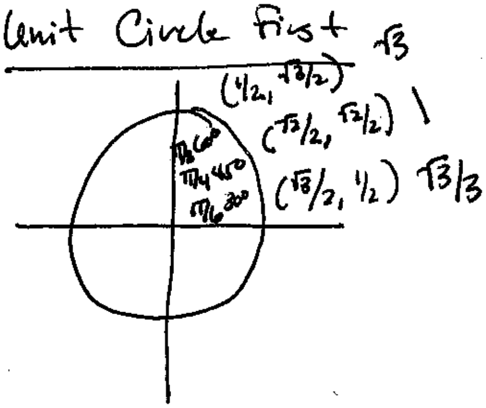
① Unit Circle first



GSE PreCalculus
Guided Notes

Sum/Difference Identities

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Example 1: Use the sum and difference identities to find the exact value of sin/cos/tan

$15° = (45° - 30°)$

45°: $(\sqrt{2}/2, \sqrt{2}/2)$ cos sin, Tan 1

30: $(\sqrt{3}/2, 1/2)$ cos sin, Tan $\sqrt{3}/3$

a. $\sin 15°$

$\sin 15° = \sin(45° - 30°)$

$= \sin 45 \cos 30 - \cos 45 \sin 30$

$(\sqrt{2}/2)(\sqrt{3}/2) - (\sqrt{2}/2, 1/2)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

b. $\cos 15°$

$\cos 15 = \cos(45 - 30)$

$= \cos 45 \cos 30 + \sin 45 \sin 30$

$(\sqrt{2}/2)(\sqrt{3}/2) + (\sqrt{2}/2)(1/2)$

$\sqrt{6}/4 + \cancel{\sqrt{2}/4}\ \sqrt{2}/2 = \frac{\sqrt{6}+\sqrt{2}}{4}$

c. $\tan 15°$

$$= \frac{\sin 15°}{\cos 15°}$$

$$\frac{\frac{\sqrt{6}-\sqrt{2}}{4}}{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

$$\frac{\sqrt{6}-\sqrt{2}}{\cancel{4}} \cdot \frac{\cancel{4}}{\sqrt{6}+\sqrt{2}}$$

$$\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} \cdot \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \frac{6-\sqrt{12}-\sqrt{12}+2}{6-\sqrt{12}+\sqrt{12}-2} = \frac{8-2\sqrt{12}}{6-2}$$

$$= \frac{8-4\sqrt{3}}{4} = \boxed{2-\sqrt{3}}$$

$\alpha \quad \beta \quad \alpha \quad \beta$

Example 2: Rewrite the expression using sin, cos, or tan: $\sin 340° \cos 50° - \cos 340° \sin 50°$

$\sin(\alpha-\beta)$ $\quad$ $\sin(340-50)$

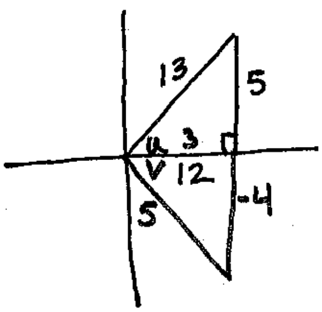
$\sin(290°)$

Example 3: Find the exact value of the trig function given:

$\sin u = \frac{5}{13} \quad 0 < u < \frac{\pi}{2}$ Q1, +,+

$\cos v = \frac{3}{5} \quad \frac{3\pi}{2} < v < 2\pi$ Q4, +,-

Find $\cos(u+v) = \cos u \cos v - \sin u \sin v$



$5^2 + b^2 = 13^2$

$25 + b^2 = 169$

$b^2 = 144$

$b = 12$

$(12/13)(3/5) - (5/13)(-4/5)$

$= 36/65 + 20/65$

$= 56/65$

$\alpha \quad \beta \quad \alpha \quad \beta$

Example 4: Verify $\sin(x+\frac{\pi}{3}) + \sin(x-\frac{\pi}{3}) = \sin x$

$\pi/3 \quad (1/2, \sqrt{3}/2)$
cos sin

$\sin x \cos \pi/3 + \cos x \sin \pi/3 + \sin x \cos \pi/3 - \cos x \sin \pi/3 = \sin x$

$\sin x(1/2) + \cancel{\cos x(\sqrt{3}/2)} + \sin x(1/2) - \cancel{\cos x(\sqrt{3}/2)} = \sin x$

$1/2 \sin x + 1/2 \sin x$

$1 \sin x = \sin x$ ✓