GSE PreCalculus: Unit 5: Trig Identities
Test 5A Review

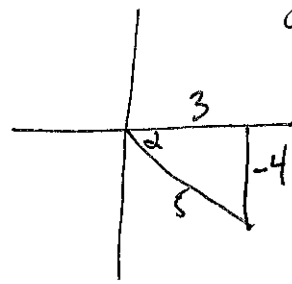
Name Answer Key
Date Fall 2017 Day ____

I. Review all Identities – Reciprocal, Ratio, PT Identities, Even/Odd (Sum & Difference will be given)

II. Know Unit Circle and how to find exact values.

Given that α and β **are in quadrant 4 and** $\sin\alpha = -\frac{4}{5}$ and $\cos\beta = \frac{15}{17}$, **Draw triangles**

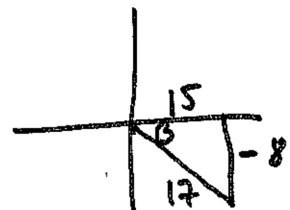
$a^2 + (-4)^2 = 5^2$
$a^2 = 9$
$a = 3$



Find:

1. $\cos(\alpha) = 3/5$

2. $\sin(\beta) = -8/17$



3. $\sin(\alpha+\beta)$ $\sin\alpha\cos\beta + \cos\alpha\sin\beta$
$(-4/5)(15/17) + (3/5)(-8/17)$
$\frac{-60}{85} + \left(\frac{-24}{85}\right) = \boxed{\frac{-84}{85}}$

4. $\cos(\alpha-\beta)$ $\cos\alpha\cos\beta + \sin\alpha\sin\beta$
$\left(\frac{3}{5}\right)\left(\frac{15}{17}\right) + (-4/5)(-8/17)$ $\quad \frac{45}{85} + \frac{32}{85} = \boxed{\frac{77}{85}}$

$(15)^2 + b^2 = (17)^2$
$225 + b^2 = 289$
$b^2 = 64$
$b = 8$

* use tan formula then take the reciprocal
5. $\cot(\alpha-\beta)$ $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{(-4/3) - (-8/15)}{1+\left(\frac{-4}{3}\right)\left(\frac{-8}{15}\right)} = \frac{-\frac{4}{5}}{77/45}$

$\frac{-4}{5} \cdot \frac{45}{77} = \frac{-36}{77}$ $\quad \boxed{\cot(\alpha-\beta) = \frac{-77}{36}}$

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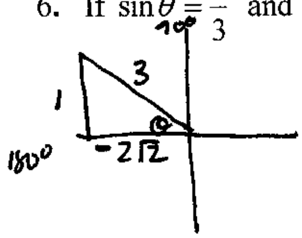
6. If $\sin\theta = \frac{1}{3}$ and $90° < \theta < 180°$, then find the value of $\sec\theta$

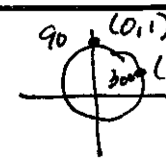


$1^2 + b^2 = 3^2$
$1 + b^2 = 9$
$b^2 = 8$
$b = 2\sqrt{2}$

$\cos\theta = \frac{-2\sqrt{2}}{3}$

$\sec\theta = \frac{3}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{-3\sqrt{2}}{4}}$

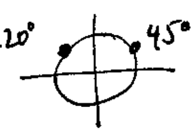
Use sum/difference formulas to find the exact value of the following:



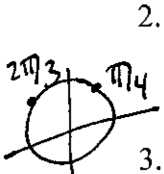
1. $\sin 60° = \sin(\underset{\alpha}{90°} - \underset{\beta}{30°})$
$\sin\alpha\cos\beta - \cos\alpha\sin\beta$
$\sin 90\cos 30 - \cos 90 \sin 30$
$(1)\left(\frac{\sqrt{3}}{2}\right) - (0)\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\sqrt{3}}{2}}$

$90°\ \alpha$
$(0,1)$
$30°\ \beta$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

2. $\cos 75° = \cos(120° - 45°)$
$\cos 120\cos 45 + \sin 120 \sin 45$
$(-1/2)(\sqrt{2}/2) + (\sqrt{3}/2)(\sqrt{2}/2) = \frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$



$120\ \alpha \quad 45\ \beta$
$(-1/2, \sqrt{3}/2)\ \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$



3. $\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$
$\sin 2\pi/3 \cos \pi/4 + \cos 2\pi/3 \sin \pi/4$
$\left(\frac{\sqrt{3}}{2}\right)(\sqrt{2}/2) + (-1/2)(\sqrt{2}/2)$

$2\pi/3\ \alpha \quad \pi/4\ \beta$
$(-1/2, \sqrt{3}/2) \quad (\sqrt{2}/2, \sqrt{2}/2)$

$\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

Write as the sin, cos, or tan of a single angle.

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* Write one expression! *

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1. $\sin 70^\circ \cos 40^\circ - \cos 70^\circ \sin 40^\circ$ $\sin(70-40) = \sin 30^\circ$

2. $\cos 210^\circ \cos 80^\circ + \sin 210^\circ \sin 80^\circ$ $\cos(210-80) = \cos 130^\circ$

3. $\dfrac{\tan 43^\circ - \tan 13^\circ}{1+\tan 43^\circ \tan 13^\circ}$ $\tan(43-13) = \tan 30^\circ$

Verify the following. Hint: Sum + Diff twice

1. $\sin(x+y)+\sin(x-y) = 2\sin x\cos y$

$\sin x\cos y + \cancel{\cos x\sin y} + \sin x\cos y - \cancel{\cos x\sin y}$

$\sin x\cos y + \sin x\cos y$

$2\sin x\cos y = 2\sin x\cos y$ ✓

* Diff of cubes $(a^3-b^3) = (a-b)(a^2+ab+b^2)$

2. $\dfrac{\tan^3\theta - 1}{\tan\theta - 1} = \tan^2\theta + \tan\theta + 1$

$\dfrac{\cancel{(\tan\theta-1)}(\tan^2\theta+\tan\theta+1)}{\cancel{\tan\theta-1}}$

$\tan^2\theta + \tan\theta + 1 = \tan^2\theta + \tan\theta + 1$ ✓

PT Identity! Hint: DOTS

3. $\sec^4 x - \tan^4 x = 1 + 2\tan^2 x$

$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$

$(1 + \cancel{\tan^2 x} - \cancel{\tan^2 x})(1+\tan^2 x + \tan^2 x)$

$(1)(1+2\tan^2 x)$

$1 + 2\tan^2 x = 1 + 2\tan^2 x$ ✓

PT Identity

4. $\cos^2 x(1+\tan^2 x) = 1$

$\cos^2 x(\sec^2 x)$

$\cancel{\cos^2 x}\left(\frac{1}{\cancel{\cos^2 x}}\right)$

$1 = 1$ ✓

Rewrite

5. $1+\sec^2\theta\sin^2\theta = \sec^2\theta$

$1 + \left(\frac{1}{\cos^2\theta}\right)\sin^2\theta$

Rewrite

$1 + \frac{\sin^2\theta}{\cos^2\theta}$ $1+\tan^2\theta$

$\sec^2\theta = \sec^2\theta$ ✓

Common Denominator $(1-\cos x)(1+\cos x)$

6. $\frac{1+\cos x}{1+\cos x}\cdot\frac{1}{1-\cos x} - \frac{1}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x} = \cancel{2\csc x\cot x}$

Distribute

$\dfrac{1+\cos x - (1-\cos x)}{(1-\cos x)(1+\cos x)} = \dfrac{\cancel{1}+\cos x - \cancel{1}+\cos x}{1-\cos^2 x}$

Multiply

$\dfrac{2\cos x}{\sin^2 x} = 2\cdot\dfrac{\cos x}{\sin x}\cdot\dfrac{1}{\sin x}$

$2\csc x\cot x$ ✓

7. $\dfrac{\sin x}{\sin x - \cos x} = \boxed{\dfrac{1}{1-\cot x}}$ = work the Right side

Common Denominator

$\dfrac{1}{1-\frac{\cos x}{\sin x}}$

$\dfrac{1}{\frac{\sin x - \cos x}{\sin x}}$

KCF

$1\cdot\dfrac{\sin x}{\sin x - \cos x}$

$\dfrac{\sin x}{\sin x - \cos x} = \dfrac{\sin x}{\sin x - \cos x}$ ✓

Sum + Diff

8. $\cos\left(\theta - \frac{\pi}{3}\right) + \cos\left(\theta+\frac{\pi}{3}\right) = \cos\theta$

$\pi/3$ $\pi/6$

$\pi/3\ (1/2, \sqrt{3}/2)$

$\pi/6\ (\sqrt{3}/2, 1/2)$

$\cos\theta\cos\pi/3 + \sin\theta\sin\pi/3$

$+ \cos\theta\cos\pi/3 - \sin\theta\sin\pi/3$

$1/2\cos\theta + \cancel{\sqrt{3}/2\sin\theta} + 1/2\cos\theta - \cancel{\sqrt{3}/2\sin\theta}$

$1/2\cos\theta + 1/2\cos\theta$

$= 1\cos\theta = \cos\theta$ ✓