GSE PreCalculus: Unit 5 – Verifying Trig Identities

WS 5.4: Sum/Difference Identities

Name Key

Date

Find the exact value of each expression.

1. $\cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$ (α, β)

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\pi/4)\cos(\pi/3) - \sin(\pi/4)\sin(\pi/3)$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

2. $\cos\left(\frac{3\pi}{4}+\frac{5\pi}{6}\right)$ (α, β)

$\cos(3\pi/4)\cos(5\pi/6) - \sin(3\pi/4)\sin(5\pi/6)$

$\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

Write the expression as sine, cosine, or tangent.

3. $\cos 25°\cos 15° - \sin 25°\sin 15°$ (α, β)

$\cos(\alpha+\beta)$

$\cos(25+15) = \cos 40$

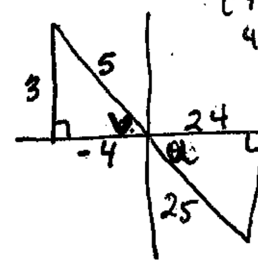
4. $\sin 140°\cos 50° + \cos 140°\sin 50°$

$\sin(\alpha+\beta)$

$\sin(140+50) = \sin 190$

Find the exact value of the trig function given that $\sin u = -\frac{7}{25}$ $\frac{3\pi}{2}<u<2\pi$ (4th $+, -$)

$\cos v = -\frac{4}{5}$ $\frac{\pi}{2} < v < \pi$ (2nd $-, +$)



$(-7)^2 + b^2 = 25^2$

$49 + b^2 = 625$

$b^2 = 576$

$b = 24$

5. cos (u + v) $= \cos u\cos v - \sin u\sin v$

$\left(\frac{24}{25}\right)\left(-\frac{4}{5}\right) - \left(-\frac{7}{25}\right)\left(\frac{3}{5}\right)$

$\frac{-96}{125} + \frac{21}{125} = \frac{-75}{125}$

$= \boxed{-\frac{3}{5}}$

6. sin (u + v)

$\sin u\cos v + \cos u\sin v$

$\left(-\frac{7}{25}\right)\left(-\frac{4}{5}\right) + \left(\frac{24}{25}\right)\left(\frac{3}{5}\right)$

$\frac{28}{125} + \frac{72}{125}$

$= \boxed{\cancel{\frac{-44}{125}}} = \frac{100}{125} = \boxed{\frac{4}{5}}$

7. tan (v – u)

$\frac{\tan v - \tan u}{1 + \tan u\tan v}$

$\frac{\left(-\frac{3}{4}\right) - \left(-\frac{7}{24}\right)}{1 + \left(-\frac{7}{24}\right)\left(-\frac{3}{4}\right)}$

$\frac{-\frac{11}{24}}{\frac{39}{32}}$

$-\frac{11}{24} \cdot \frac{32}{39} = \boxed{\frac{-44}{117}}$

8. sec (u + v)

$\frac{1}{\cos}$ so, $= \boxed{-\frac{5}{3}}$

9. cot (u + v)

$\tan(u+v) = \frac{\tan u + \tan \beta}{1 - \tan u \tan \beta}$

$\frac{\left(-7/24\right) + \left(-\frac{3}{4}\right)}{1 - \left(-7/24\right)\left(-3/4\right)}$

$= \frac{\frac{-25}{24}}{\frac{25}{32}}$ $\quad \frac{-25}{24} \cdot \frac{32}{25}$

$= -4/3$, so

$\cot = \boxed{-3/4}$

Verify the identities.

8. $\sin(3\pi - x) = \sin x$

$\alpha \quad \beta$

$\sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\sin 3\pi \cos x - \cos 3\pi \sin x = \sin x$

$\cancel{0 \cos x} - (-1)\sin x$

$+ 1 \sin x$

$\sin x = \sin x$

9. $\sin\left(\dfrac{\pi}{2} + x\right) = \cos x$

$\sin \pi/2 \cos x + \cos \pi/2 \sin x$

$1 \cdot \cos x + \cancel{(0)\sin x}$

$\cos x = \cos x$ ✓

10. $\sin(x+y) + \sin(x-y) = 2\sin x\cos y$

$\sin x\cos y + \cancel{\cos x\sin y} + \sin x\cos y - \cancel{\cos x\sin y}$

$2\sin x\cos y = 2\sin x\cos y$ ✓

11. $\cos x + \dfrac{\sin^2 x}{\cos x} = \sec x$

$\dfrac{\cos x}{\cos x} \cdot \dfrac{\cos x}{1}$

$\dfrac{\cos^2 x}{\cos x} + \dfrac{\sin^2 x}{\cos x}$

$\dfrac{\cos^2 x + \sin^2 y}{\cos x}$

$\dfrac{1}{\cos x} = \sec x = \sec x$ ✓

12. $\cos x + \tan x \sin x = \sec x$

$\cos x + \dfrac{\sin x}{\cos x} \cdot \sin x = \sec x$

$\dfrac{\cos x}{\cos x} \cdot \dfrac{\cos x}{1} + \dfrac{\sin^2 x}{\cos x}$

$\dfrac{\cos^2 x + \sin^2 x}{\cos x} = \dfrac{1}{\cos x} = \sec x$ ✓

13. $\sin^3 x(1 - 2\cos^2 x + \cos^4 x) = \sin^7 x$

$\sin^3 x\ (\cancel{\cos^4 x - 2\cos^2 x + 1})$

$\cancel{\sin^3 x\ (\cos^2 x - 1)(\cos^2 x - 1)}$

$\sin^3 x\ (1 - \cos^2 x)(1 - \cos^2 x)$

$\sin^3 x\ (\sin^2 x)(\sin^2 x)$

$\sin^7 x = \sin^7 x$ ✓

14. $\dfrac{\cos x}{1 - \sin x} = \dfrac{1 + \sin x}{\cos x}$

$\dfrac{\cos x}{1 - \sin x} \cdot \dfrac{1 + \sin x}{1 + \sin x}$

$\dfrac{\cos x\,(1 + \sin x)}{1 + \sin x - \sin x - \sin^2 x}$

$\dfrac{\cos x\,(1 + \sin x)}{1 - \sin^2 x}$

$\dfrac{\cancel{\cos x}\,(1 + \sin x)}{\cos^2 x}$

$\dfrac{1 + \sin x}{\cos x} = \dfrac{1 + \sin x}{\cos x}$ ✓