

Verify the identity algebraically

1. $\csc\theta \tan\theta = \sec\theta$

$$\left(\frac{1}{\sin\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$\frac{1}{\cos\theta} =$$

$$\sec\theta = \sec\theta \checkmark$$

2. $\cos\theta \sec\theta - \cos^2\theta = \sin^2\theta$

$$\cos\theta \left(\frac{1}{\cos\theta}\right) - \cos^2\theta$$

$$\cos\theta \left(\frac{1}{\cos\theta} = \frac{\cos\theta}{1}\right)$$

$$1 - \cos^2\theta$$

$$\sin^2\theta = \sin^2\theta \checkmark$$

3. $\frac{\csc\theta}{\sec\theta} + \frac{\cos\theta}{\sin\theta} = 2\cot\theta$

Hint: simplify $\frac{\csc\theta}{\sec\theta}$ in terms of $\sin\theta, \cos\theta$

$$\frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} + \frac{\cos\theta}{\sin\theta}$$

$$\frac{\cos\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{2\cos\theta}{\sin\theta}$$

$$= 2\cot\theta = 2\cot\theta$$

$$\frac{1}{\sin\theta} \cdot \frac{\cos\theta}{1} + \frac{\cos\theta}{\sin\theta}$$

4. $\frac{\sec^2\theta - \tan^2\theta + \tan\theta}{\sec\theta} = \cos\theta + \sin\theta$ Very Challenging!!!

$$\frac{(1 + \tan^2\theta) - \tan^2\theta + \tan\theta}{\sec\theta}$$

$$\sec\theta$$

$$\frac{1 + \tan\theta}{\sec\theta} + \frac{\sin\theta}{\cos\theta}$$

$$\frac{\cos\theta \tan\theta}{\cos\theta} \cdot \frac{\cos\theta}{1}$$

$$\frac{1}{\cos\theta}$$

$$\frac{1}{\cos\theta}$$

$$\cos\theta + \sin\theta = \cos\theta + \sin\theta$$

5. $\sin t \csc t = 1$

$$\sin t \cdot \frac{1}{\sin t}$$

$$1 = 1$$

6. $\tan y \cot y = 1$

$$\left(\frac{\sin y}{\cos y}\right)\left(\frac{\cos y}{\sin y}\right)$$

$$1 = 1$$

7. $\frac{\csc^2 x}{\cot x} = \csc x \sec x$

$$\left(\frac{1}{\sin^2 x}\right)\left(\frac{\sin x}{\cos x}\right)$$

$$\left(\frac{1}{\sin x}\right)\left(\frac{1}{\cos x}\right) \csc x \sec x$$

9. $\cos^2\beta - \sin^2\beta = 2\cos^2\beta - 1$

$$\cos^2\beta - (1 - \cos^2\beta)$$

$$\cos^2\beta - 1 + \cos^2\beta$$

$$2\cos^2\beta - 1 = 2\cos^2\beta - 1 \checkmark$$

8. $\cot^2 y (\sec^2 y - 1) = 1$ Hint: rewrite $\sec^2 y - 1$

$$\cot^2 y (\tan^2 y) = 1$$

$$\left(\frac{\cos^2 y}{\sin^2 y}\right)\left(\frac{\sin^2 y}{\cos^2 y}\right) = 1$$

10. $\tan^2\theta + 6 = \sec^2\theta + 5$

$$(\sec^2\theta - 1) + 6$$

$$\sec^2\theta + 5 = \sec^2\theta + 5$$



14.

$$\sec^4 \theta - \tan^4 \theta = 1 + 2\tan^2 \theta$$

$$(\sec^2 \theta - \tan^2 \theta)(\sec^2 \theta + \tan^2 \theta) = 1 + 2\tan^2 \theta$$

$$(\cancel{\tan^2 \theta + 1} - \cancel{\tan^2 \theta})(\tan^2 \theta + 1 + \tan^2 \theta)$$

$$1 + \tan^2 \theta + \tan^2 \theta$$

$$1 + 2\tan^2 \theta = 1 + 2\tan^2 \theta \checkmark$$

15.
$$\csc^4 \theta - \cot^4 \theta = 2\csc^2 \theta - 1$$

~~$$(\csc^2 \theta - \cot^2 \theta)(\csc^2 \theta + \cot^2 \theta) = 2\csc^2 \theta - 1$$~~

~~$$(1 + \cot^2 \theta - \cot^2 \theta)(1 + \cot^2 \theta + \cot^2 \theta)$$~~

1+

$$(\csc^2 \theta - \cot^2 \theta)(\csc^2 \theta + \cot^2 \theta)$$

$$[\csc^2 \theta - (\csc^2 \theta - 1)](\csc^2 \theta + \csc^2 \theta - 1)$$

$$\cancel{\csc^2 \theta} - \cancel{\csc^2 \theta} + 1 (2\csc^2 \theta - 1)$$

$$2\csc^2 \theta - 1 = 2\csc^2 \theta - 1 \checkmark$$