

$$\text{16. } \frac{1}{(1+\cos\theta)(1-\cos\theta)} + \frac{1}{(1-\cos\theta)(1+\cos\theta)} = 2 \csc^2\theta$$

$$\frac{1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)} + \frac{1+\cos\theta}{(1-\cos\theta)(1+\cos\theta)}$$

$$\frac{\overbrace{1+1}^{1+1}}{\cancel{1-\cos\theta} + \cancel{1+\cos\theta}}}{(1-\cos\theta)(1+\cos\theta)}$$

$$\frac{2}{1-\cos^2\theta}$$

$$\frac{2}{\sin^2\theta}$$

$$2 \cdot \frac{1}{\sin^2\theta}$$

$$2 \csc^2\theta = 2 \csc^2\theta \quad \checkmark$$

11. $2 - \csc^2 z = 1 - \cot^2 z$

Hint: rewrite $\csc^2 z$ as $1 + \cot^2 z$

$$2 - (1 + \cot^2 z)$$

$$2 - 1 - \cot^2 z$$

$$1 - \cot^2 z = 1 - \cot^2 z \checkmark$$

13. $\frac{\csc \theta}{\sin \theta} - \frac{\cot \theta}{\tan \theta} = 1$

$$\frac{1}{\sin \theta} - \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta}}$$

$$\left(\frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\sin \theta} \right) - \left(\frac{\cos \theta}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta} \right)$$

$$\frac{1}{\sin^2 \theta} - \frac{\cos^2 \theta}{\sin^2 \theta} = 1 \checkmark$$

15. $\csc^4 \theta - \cot^4 \theta = 2 \csc^2 \theta - 1$

Hint: factor as a difference of two squares

$$x^4 - y^4 = (x^2 - y^2)(x^2 + y^2)$$

12. $\cos x + \sin x \tan x = \sec x$

$$\cos x + \sin x \left(\frac{\sin x}{\cos x} \right)$$

$$\cos x \cdot \frac{\cos x + \sin^2 x}{\cos x}$$

$$\frac{\cos^2 x + \sin^2 x}{\cos x}$$

$$\frac{1}{\cos x} = \sec x \checkmark$$

14. $\sec^4 \theta - \tan^4 \theta = 1 + 2 \tan^2 \theta$

Hint: factor as a difference of two squares

$$x^4 - y^4 = (x^2 - y^2)(x^2 + y^2)$$

16. $\frac{1}{1 + \cos \theta} + \frac{1}{1 - \cos \theta} = 2 \csc^2 \theta$

