GSE PreCalculus Name Avey's Key

Test 5B Review: Trig Identities

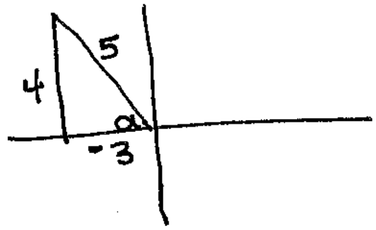
Given that α and β **are in quadrant 2 and** $\sin\alpha = \frac{4}{5}$ and $\cos\beta = -\frac{15}{17}$**, find:**

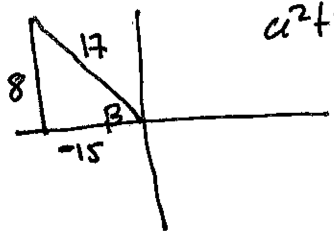
$a^2 + (-15)^2 = (17)^2$

$a^2 + 225 = 289$

$a^2 = 64$

$a = 8$





1. $\cos\alpha$ $-3/5$

2. $\sin\beta$ $8/17$

Double ∠s

3. $\sin(2\alpha) = 2\sin\alpha\cos\alpha$

$2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = \boxed{-24/25}$

4. $\cos(2\beta) = \cos^2\beta - \sin^2\beta$

$(-15/17)^2 - (8/17)^2 = \boxed{161/289}$

5. $\tan(2\beta) = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2\left(\frac{-8}{15}\right)}{1-\left(-8/15\right)^2} = \frac{-16/15}{161/225} = \boxed{-240/161}$

6. $\cos(\alpha - \beta)$

$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\left(\frac{-3}{5}\right)\left(\frac{-15}{17}\right) + \left(\frac{4}{5}\right)\left(\frac{8}{17}\right) = \frac{45}{85} + \frac{32}{85} = \boxed{\frac{77}{85}}$

157.5

cos −

Use half angle formulas to solve the following

15 sin +

7. $\cos 157.5° =$ $\cos\frac{315}{2} = \sqrt{\frac{1+\cos 315}{2}}$

$2(157.5) = 315$

$= \sqrt{\frac{1+\sqrt{2}/2}{2}}$

Pull it out

$1 + \frac{\sqrt{2}}{2}$

Common denominator $\frac{2}{2} + \frac{\sqrt{2}}{2}$ $\frac{\frac{2+\sqrt{2}}{2}}{2}$ ← KCF $\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}$

$= \sqrt{\frac{2+\sqrt{2}}{4}}$

$= \boxed{-\frac{\sqrt{2+\sqrt{2}}}{2}}$

8. $\sin 15° =$ $\sin\frac{30}{2} = \sqrt{\frac{1-\cos 30}{2}}$

$2(15) = 30$

$= \sqrt{\frac{1-\sqrt{3}/2}{2}}$

$1 - \frac{\sqrt{3}}{2}$

$\frac{2}{2} - \frac{\sqrt{3}}{2} = \frac{\frac{2-\sqrt{3}}{2}}{2}$ $\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}$

$= \sqrt{\frac{2-\sqrt{3}}{4}}$

$= \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$

Solve.

9. $2\sin^2 x = 2 + \cos x$

$2\sin^2 x - 2 - \cos x = 0$

$2(1-\cos^2 x) - 2 - \cos x$

$\cancel{2} - 2\cos^2 x - \cancel{2} - \cos x$

$\cos x(-2\cos x - 1) = 0$

$\cos x = 0$ $\quad -2\cos x - 1 = 0$

$\boxed{90°}$ $\quad -2\cos x = 1$

$\cos x = -1/2$

$\boxed{120°}$

10. $2\sin\alpha\cos\alpha = \sin\alpha$

$2\sin\alpha\cos\alpha - \sin\alpha = 0$

$\sin\alpha(2\cos\alpha - 1) = 0$

$\sin\alpha = 0$ $\quad 2\cos\alpha - 1 = 0$

$\boxed{0°}$ $\quad \frac{2\cos\alpha}{2} = \frac{1}{2}$

$\cos\alpha = 1/2$

$\boxed{60°}$

11. $\sin^2 x - 3\cos x = 3$

$\sin^2 x - 3\cos x - 3 = 0$

$1 - \cos^2 x - 3\cos x - 3 = 0$

$-\cos^2 x - 3\cos x - 2 = 0$

$\frac{-1(\cos^2 x + 3\cos x + 2)}{-1} = \frac{0}{-1}$

$\cos^2 x + 3\cos x + 2 = 0$

$(\cos x + 1)(\cos x + 2) = 0$

$\cos x + 1 = 0$ $\quad \cos x + 2 = 0$

$\cos x = -1$ $\quad \cancel{\cos x = -2}$

$\boxed{180°}$

12. $2\sin^2 x = 9\sin x + 5$

$2\sin^2 x - 9\sin x - 5 = 0$

$(\sin x - 5)(2\sin x + 1)$

$\sin x - 5 = 0$ $\quad 2\sin x + 1 = 0$

$\cancel{\sin x = 5}$ $\quad 2\sin x = -1$

$\sin x = -1/2$

$\boxed{x = -30°}$

13. $\sin^2\beta - \sin\beta = 0$

$\sin\beta(\sin\beta - 1) = 0$

$\sin\beta = 0$ $\quad \sin\beta - 1 = 0$

$\boxed{0°}$ $\quad \sin\beta = 1$

$\boxed{90°}$

** Answers above are not on the [0, 360] **
just found the inverse!

Verify the following. $\boxed{\text{Sum + diff formula}}$

14. $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

$\sin x\cos y+\cancel{\cos x\sin y}+\sin x\cos y-\cancel{\cos x\sin y}$

$2\sin x\cos y = 2\sin x\cos y$ ✓

15. $\dfrac{\sin x}{\sin x-\cos x}=\dfrac{1}{1-\cot x}=\dfrac{1}{1-\frac{\cos x}{\sin x}}=\dfrac{1}{\frac{\sin x}{\sin x}-\frac{\cos x}{\sin x}}$

$=\dfrac{1}{\frac{\sin x-\cos x}{\sin x}} \quad 1\cdot\dfrac{\sin x}{\sin x-\cos x}$

✓ $=\dfrac{\sin x}{\sin x-\cos x}$

$\boxed{\text{DOTS}}$ $\boxed{\text{substitute}}$

16. $\sec^4 x-\tan^4 x=1+2\tan^2 x$

$(\sec^2x-\tan^2x)(\sec^2x+\tan^2x)$

$(\cancel{\tan^2x}+1-\cancel{\tan^2x})(\tan^2x+1+\tan^2x)$

$(1)(1+2\tan^2x)=1+2\tan^2x$ ✓

16. $\cos^2 x(1+\tan^2 x)=1$

$\cos^2x(\sec^2x)$

$\cancel{\cos^2x}\left(\dfrac{1}{\cancel{\cos^2x}}\right)$ $\quad 1=1$ ✓

$\boxed{\text{Double Angle}}$

17. $\csc 2\theta=\dfrac{\csc\theta}{2\cos\theta}$

$\dfrac{1}{\sin 2\theta}=\dfrac{1}{2\sin\theta\cos\theta}=\dfrac{1}{2}\cdot\boxed{\dfrac{1}{\sin\theta}}\cdot\dfrac{1}{\cos\theta}$

$=\dfrac{1}{2}\cdot\csc\theta\cdot\dfrac{1}{\cos\theta}=\dfrac{\csc\theta}{2\cos\theta}$

18. $\sec 2\theta=\dfrac{\sec^2\theta}{2-\sec^2\theta}=\dfrac{\frac{1}{\cos^2\theta}}{2-\frac{1}{\cos^2\theta}}=\dfrac{\frac{1}{\cos^2\theta}}{\frac{2\cos^2\theta-1}{\cos^2\theta}}$

$\dfrac{1}{\cancel{\cos^2\theta}}\cdot\dfrac{\cancel{\cos^2\theta}}{2\cos^2\theta-1}=\dfrac{1}{2\cos^2\theta-1}$

$=\sec 2\theta$ ✓

$\boxed{\text{DOTS}}$

19. $\cos^4 x-\sin^4 x=\cos 2x$

$(\cos^2x-\sin^2x)(\cos^2x+\sin^2x)$

$(\cos^2x-\sin^2x)(1)$

$\cos 2x=\cos 2x$ ✓

$\boxed{\text{Expand, X}}$

20. $(\sin x+\cos x)^2=1+\sin 2x$

$(\sin x+\cos x)(\sin x+\cos x)$

$\underline{\sin^2x}+\sin x\cos x+\sin x\cos x+\underline{\cos^2x}$

$1+2\sin x\cos x$

$1+\sin 2x=1+\sin 2x$ ✓

<u>Cumulative Review from Test 1-5A:</u>

21. Identify the following conics: a. $\dfrac{(x-3)^2}{25}+\dfrac{y^2}{9}=1$ $\quad$ b. $(x+1)^2+y^2=16$

a. $\boxed{\text{Ellipse}}$ $\quad$ b. $\boxed{\text{Circle}}$

22. Multiply the following matrices: $\begin{bmatrix} x & -1 \\ 2 & 3 \end{bmatrix}\cdot\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}=\begin{bmatrix} 3x+2 & 2x-1 \\ 6-6 & 4+3 \end{bmatrix}=\begin{bmatrix} 3x+2 & 2x-1 \\ 0 & 7 \end{bmatrix}$

23. Solve the linear system: $\begin{aligned}2x+4y&=8\\ x-2y&=12\end{aligned}$

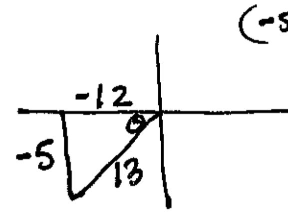
$\underset{A}{\begin{bmatrix} 2 & 4 \\ 1 & -2 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}}=\underset{B}{\begin{bmatrix} 8 \\ 12 \end{bmatrix}}$ $\quad X=A^{-1}B \quad X=\begin{bmatrix} 8 \\ -2 \end{bmatrix}$ $\quad\boxed{\begin{aligned}x&=8\\ y&=-2\end{aligned}}$

24. Find a positive co-terminal angle to: a. $\theta=-\dfrac{2\pi}{7}+2\pi=12\pi/7$ $\quad$ b. $\theta=\dfrac{\pi}{5}+2\pi=11\pi/5$

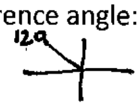
25. . If $\tan\theta=\dfrac{5}{12}$ and θ is in quadrant 3, what is the exact value of cos θ?

$\cos\theta=-12/13$

$\sqrt{(-5)^2+(-12)^2}=13$



26. Find the reference angle: a. $\theta=120°$ $\quad$ b. $\theta=315°$



a. $180-120=60°$

b. $360-315=45°$

27. Find the exact value of the following function: $\sin\left(-\dfrac{4\pi}{3}\right)=\sqrt{3}/2$

28. Evaluate: $\cos^{-1}\left(\dfrac{1}{2}\right)=60°$